

CIS6930/4930 Intro to Computational Neuroscience Spring 2006

Home Work Assignment 5: Due Monday 04/17/06 before class

The purpose of this assignment is to appreciate the complexity that can result from very simple dynamical systems.

1. (50 pts) Simulate the time course of the following parameterized class of discrete dynamical systems. The dynamics is that of a single variable $x \in [0, 1]$ that is updated by the equation $f(x_{n+1}) = a * x_n * (1 - x_n)$, where a lies in the range $[0, 4]$. Note that as long as $a \in [0, 4]$, $f(x) \in [0, 1]$ if $x \in [0, 1]$. Hence the sequence of points $x_0, f(x_0), f(f(x_0)), \dots$ never leaves the unit interval.

Your job is to plot the non-wandering (also called recurring) set for the dynamical system for a range of values of the parameter a .

Start with $a = 0$, choose a random point $x_0 \in [0, 1]$, and run the dynamical system for 5,000 points. Throw away the first 1,000 points (which are presumably the transient points until the system settles onto the non-wandering set) and plot the rest on the y-axis. Do this for several random initializations of x_0 . Next increment a by 0.01, choose a new random starting point x_0 , and do the same, and so on and so forth until you reach $a = 4.0$. (a is plotted along the x-axis).

One would assume that the non-wandering points for successive values of a should look similar. Do they? In case you find something interesting happening in a certain range for a , zoom into that range and change a by smaller increments.

2. (50 pts) Consider the 2-D phase space, \mathbb{R}^2 and the particular instantiation of the *Henon-map* given by

$$x_{n+1} = 1.245 - x_n^2 + 0.3y_n \quad y_{n+1} = x_n$$

Find and report a 2-D region in \mathbb{R}^2 that gets mapped back into itself. Now plot the attracting set of the map for points beginning in this region.