

CIS6930/4930 Intro to Computational Neuroscience Spring 2006

Home Work Assignment 4:

Due Wednesday 03/22/05 before class

1. (50 pts) Consider the following function over the range $[0, 1]$

$$f(x) = -2 \times x \quad \text{if } x \in [0, \frac{1}{3}]$$

$$f(x) = 1 \quad \text{if } x \in (\frac{1}{3}, \frac{2}{3})$$

$$f(x) = 0 \quad \text{if } x \in [\frac{2}{3}, 1]$$

Note that the function is such that

$$\int_0^1 f(x) dx = 0$$

First translate and scale uniformly the domain of the function so that it now lies on $[-\pi, +\pi]$. All future references to $f(x)$ is this scaled and translated version. Your goal will be to find an approximation of this function as a fourier series, and show the graphs of successive approximations overlaid on the actual function.

Consider the fourier basis e^{inx} for $n = -N, \dots, +N$, and the corresponding sum

$$\sum_{n=-N}^{+N} c_n e^{inx}$$

Calculate the values of c_n by numerically approximating the integral

$$\int_{-\pi}^{+\pi} f(x) e^{-inx} dx$$

, that is, by dividing the range $[-\pi, +\pi]$, into small intervals and approximating the integral as a sum.

Show graphs of how well $f(x)$ is approximated by overlaying the series over $f(x)$ for various values of N (for example, $N = 5, 10, 20, 50$).