CAP6701 Adv Graphics

Jorg Peters

- Class Web page
  https://www.cise.ufl.edu/research/SurfLab/gfxNotes/cap6701idx
- Syllabus
  https://www.cise.ufl.edu/research/SurfLab/gfxNotes/cap6701/syllabus.html
- Project 3
  https://www.cise.ufl.edu/research/SurfLab/gfxNotes/cap6701/hw3Topic.html

CAP6701 Adv Graphics

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- Motivation Modelling

  https://www.youtube.com/watch?v=vE64RBpCsEY

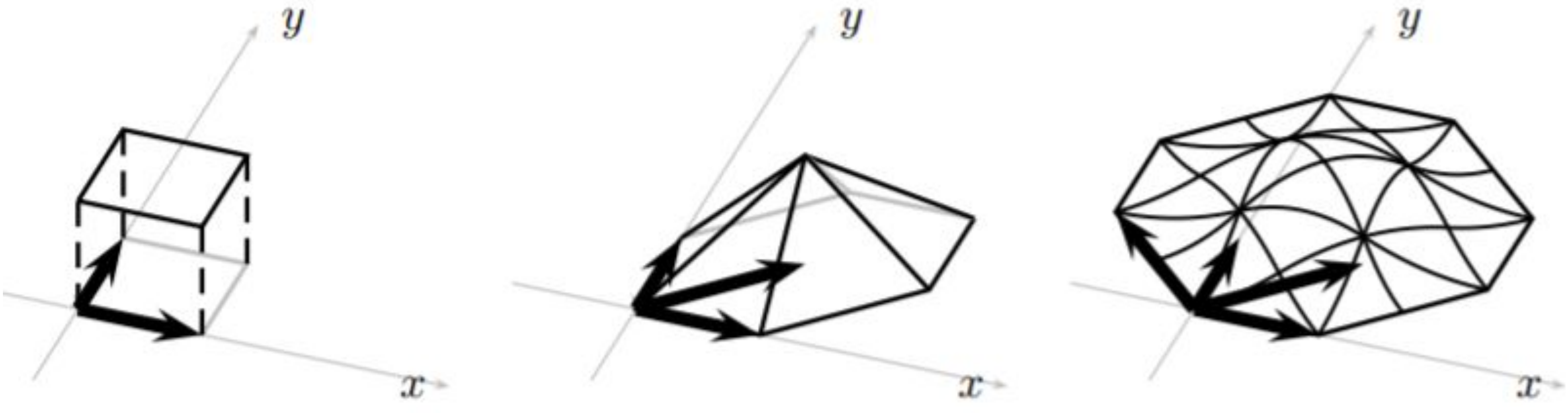
- Motivation Simulation

  https://www.youtube.com/watch?v=UnXM_EJhbRI

Assign Introtest

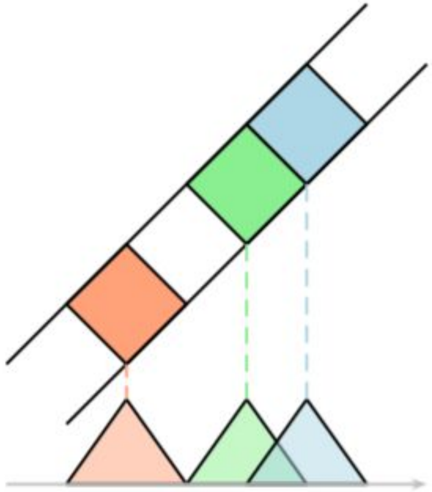
Box Splines

➢

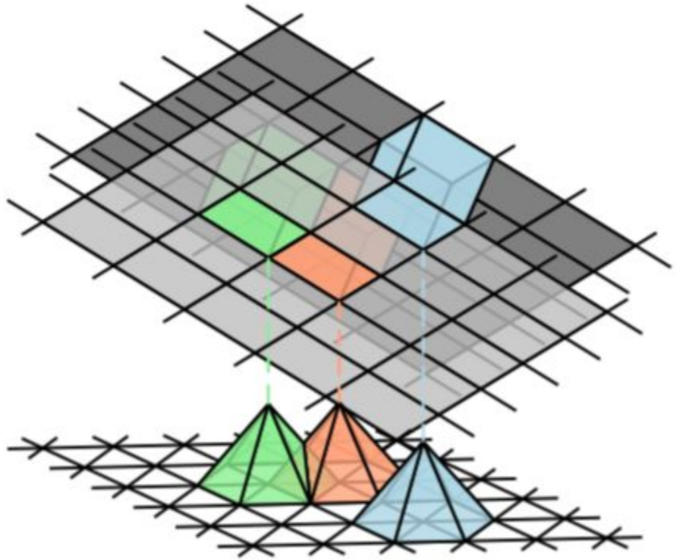


(a) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 1\end{bmatrix}$ (c) $\begin{bmatrix}1 & 0 & 1 & -1\\ 0 & 1 & 1 & 1\end{bmatrix}$

Box Splines



(a) $n = 1$



(b) $n = 2$

Box spline subdivision

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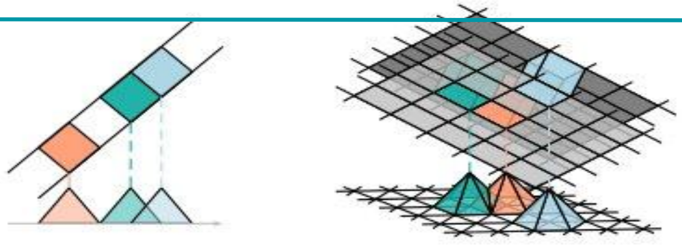
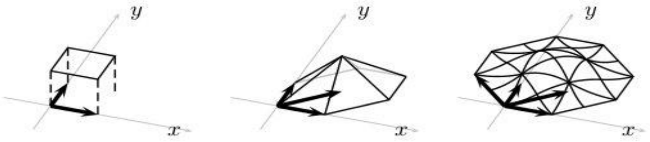
As shadows
(de Boor)



Minho Kim, Jörg Peters, *Symmetric Box-Splines on the $\mathcal{A}_n^*$ Lattice* Journal of Approximation Theory 2010.

Via convolution



B_{Q,q} = B_Q x B_q

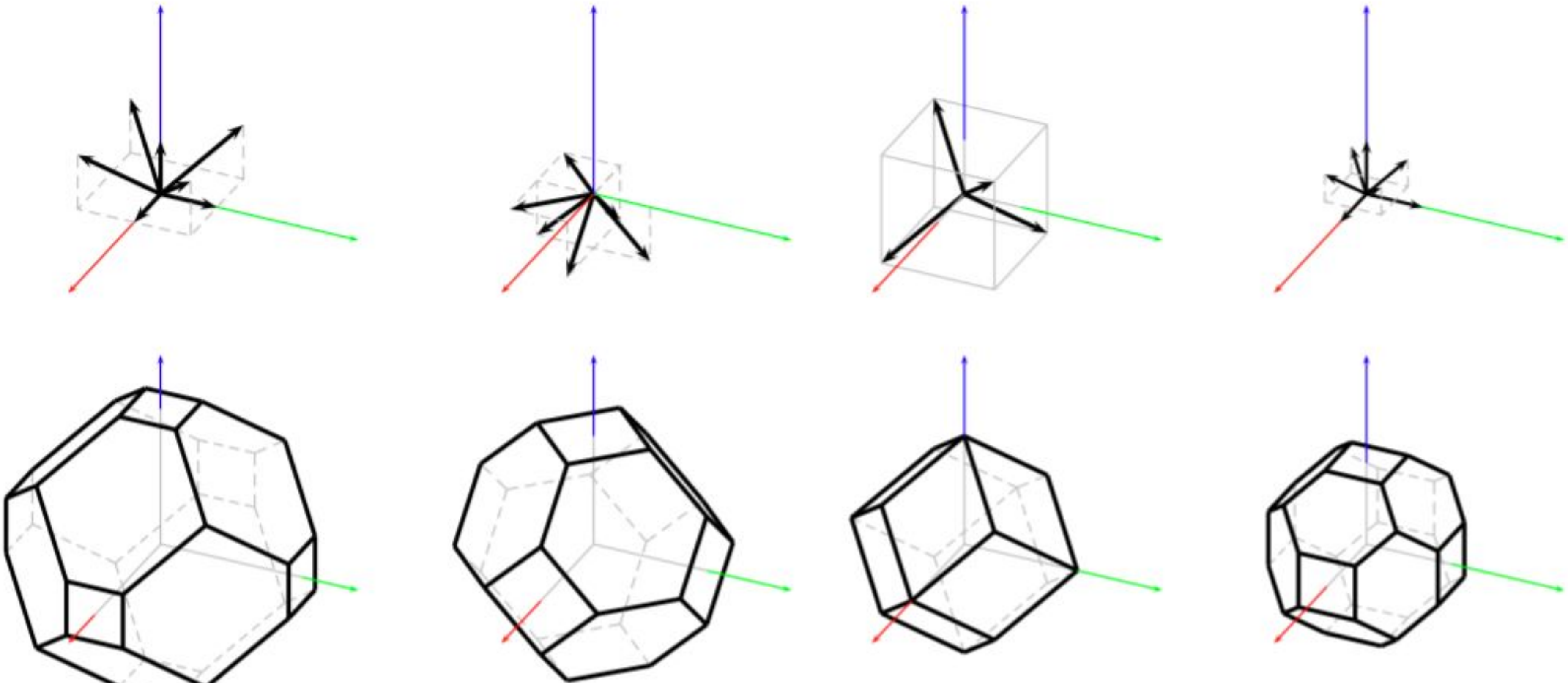
Via subdivision

$$a_{h/2}(\alpha) := \begin{cases} 2^s a_h(\alpha), \text{if } \alpha \in hZ^s \\ 0, \text{else} \end{cases}.$$

$$a_{h/2}(\alpha) \leftarrow (a_{h/2}(\alpha) + a_{h/2}(\alpha - \xi/2))/2.$$

Box spline subdivision

Box Splines (3 variables)



C1 degree 2 box spline

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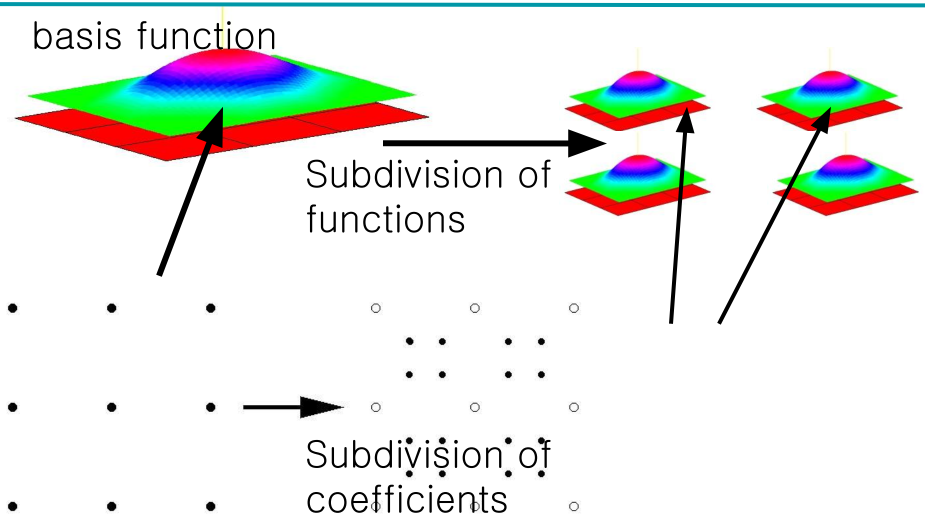

$$\frac{1}{4}\begin{matrix} 2 & 1 \\ 1 & 0 \end{matrix}$$

Subdivision Rule

$$\begin{matrix} 4a & 4b \\ 4c & 4d \end{matrix} \rightarrow \begin{matrix} a & a & b & b \\ a & a & b & b \\ c & c & d & d \\ c & c & d & d \end{matrix} \rightarrow_{1/2} \begin{matrix} . & . & . & . \\ aa & ab & bb & . \\ ac & bc & bd & . \\ cc & cd & dd & . \end{matrix} \rightarrow_{1/4} \begin{matrix} . & . & . & . \\ . & . & . & . \\ . & aabc & abbd & . \\ . & accd & bcdd & . \end{matrix}$$

Simplest subdivision

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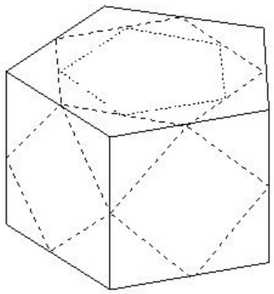
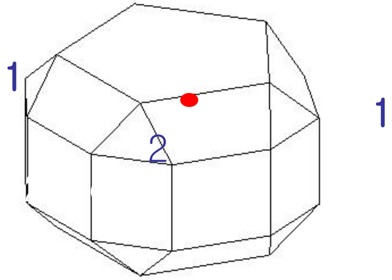
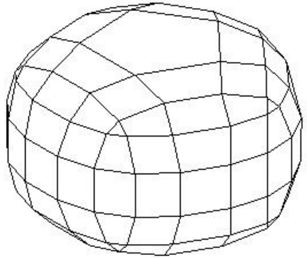
Mid-edge (1,1)/2



Connect each mid-edge to those mid-edges that share a face and a vertex





simplest = sqrt{2}subdivision!

2 steps of simplest =
box spline subdivision!

Box spline subdivision

Simplest Subdivision

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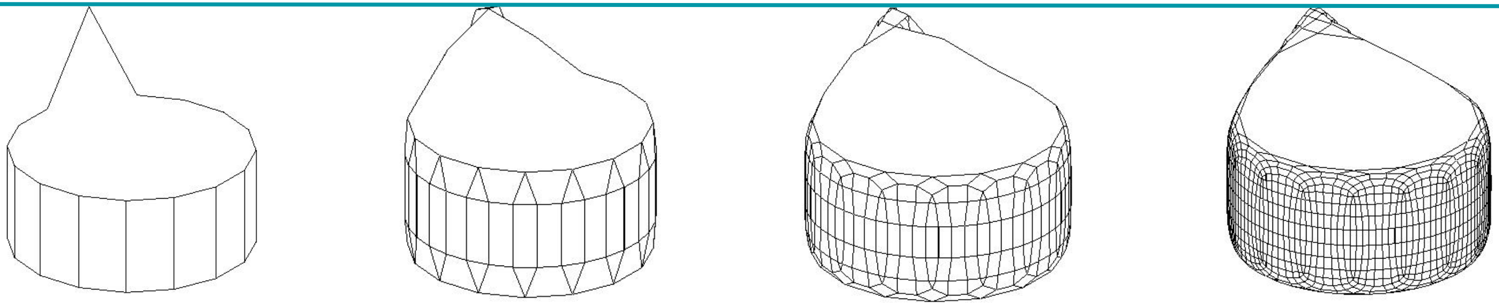
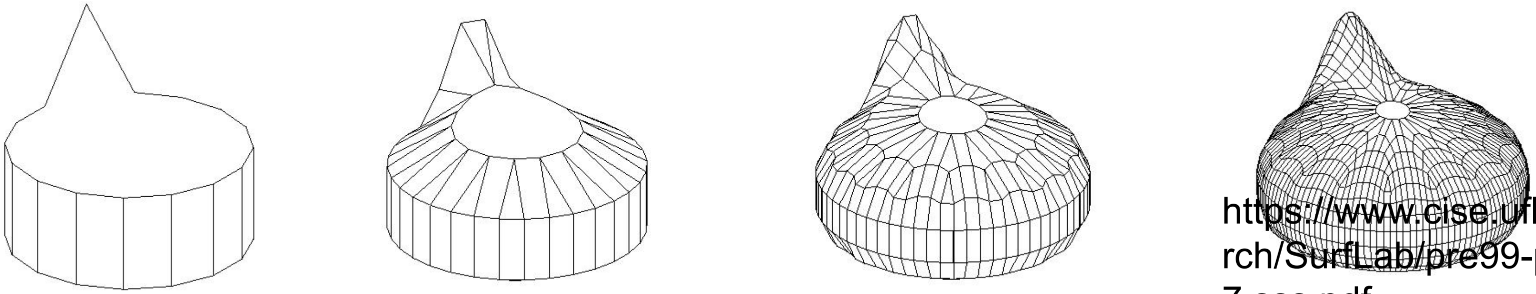


Simplest Subdivision: 8-fold eigenvalue of ¼ □ slow convergence



https://www.cise.ufl.edu/research/SurfLab/pre99-papers/9697.sss.pdf

Modified to be similar to Doo-Sabin subdivision (next)

Box spline subdivision

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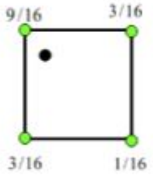
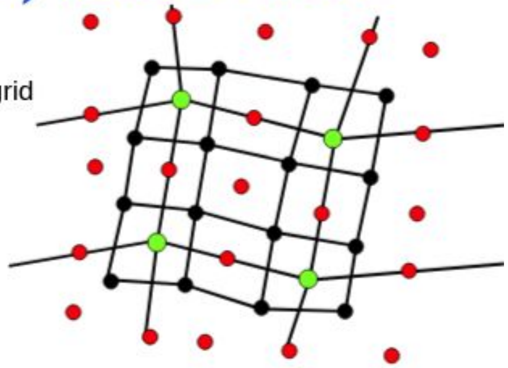
Doo-Sabin (bi-2) subdivision

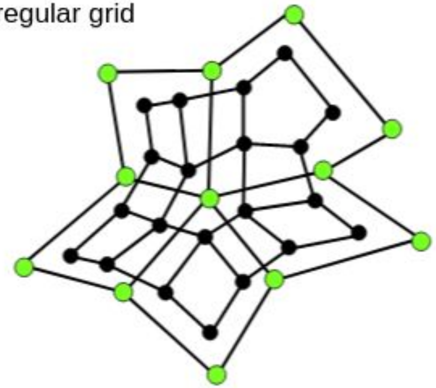
$P^{i+1} = SP^{i}$



regular grid



Irregular grid



Box spline subdivision

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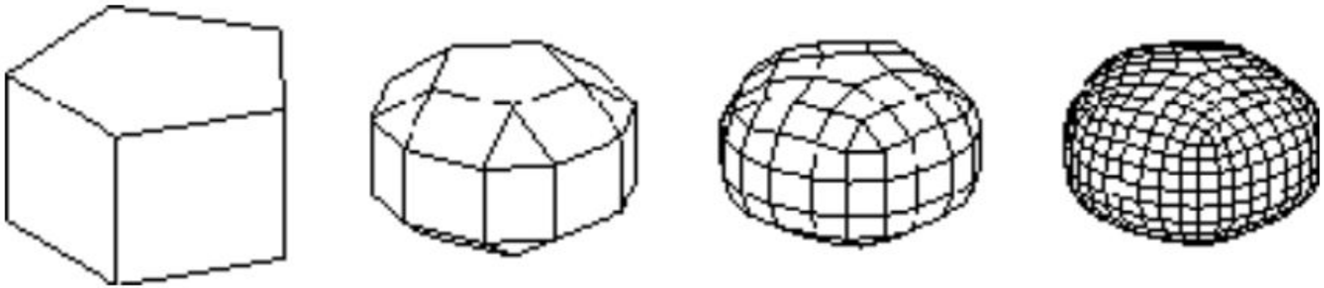
Doo-Sabin[1978]
Chaikin's corner cutting (carpenters algorithm): 1/4,3/4
Bi-2 spline

$$\begin{matrix} 1/16 & 3/16 \\ 3/16 & 9/16 \end{matrix} \quad \text{to } n\text{-sided facets.} \tag{1}$$

Doo-Sabin generalizes bi-2 spline subdivision to n-sided configuration:

$$\alpha_j := \frac{(j=0)}{4} + \frac{3 + 2\cos(2\pi\, j/n)}{4n} \tag{2}$$



Box spline subdivision

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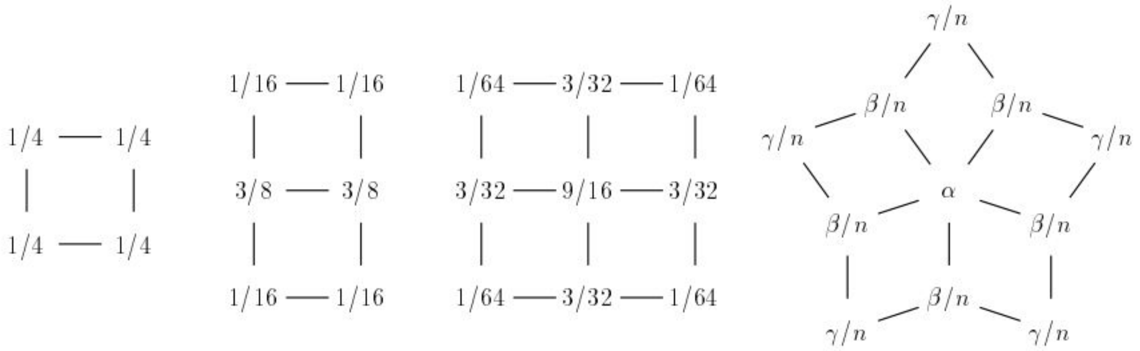
Catmull-Clark [1978]



What are the corner cutting ratios here?