BB-form(ulas)

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The derivative $\partial \mathbf{p}_n$ of a polynomial $\mathbf{p}_n$ in Bernstein form must be writable as a polynomial of one degree less in Bernstein form:

Differentiation = Differencing coefficients

$$\partial\Big(\sum_{i=0}^{n} \mathbf{c}_{n-i,i} b_{n-i,i}\Big) = \sum_{i=0}^{n-1} \mathbf{d}_{n-1-i,i} b_{n-1-i,i} \qquad \mathbf{d}_{n-i,i} := n(\mathbf{c}_{n-i-1,i+1} - \mathbf{c}_{n-i,i}).$$

$$\sum_{i=0}^{n_1} \mathbf{c}^1_{n_1-i,i} b_{n_1-i,i} * \sum_{i=0}^{n_2} \mathbf{c}^2_{n_2-i,i} b_{n_2-i,i} = \sum_{i=0}^{n} \mathbf{c}_{n-i,i} b_{n-i,i}$$

where $n = n_1 + n_2$, and

Multiplication = Collecting coefficients with equal index sums

$$\mathbf{c}_{n-i,i} = \sum_{i_1+i_2=i} \frac{\binom{n_1}{i_1}\binom{n_2}{i_2}}{\binom{n}{i}} \mathbf{c}^1_{n-i_1,i_1} \mathbf{c}^2_{n-i_2,i_2}.$$

$$\int_0^1 \sum \mathbf{c}_{n-i,i} b_{n-i,i} du = \sum \mathbf{c}_{n-i,i}/(n+1).$$

Integration = Summing coefficients