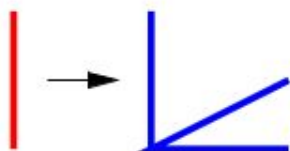
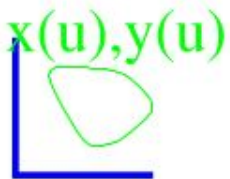
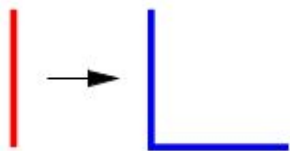
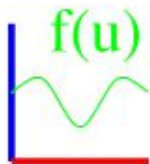
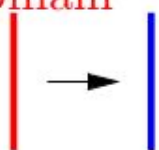


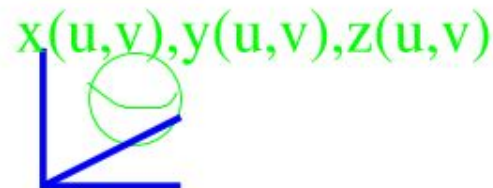
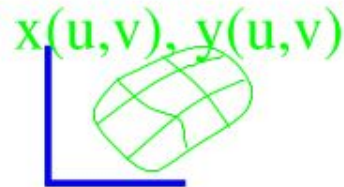
Maps and Curved Geometry

Domain, Range and Maps

^u Univariate
d Domain



^b Bivariate
d Domain



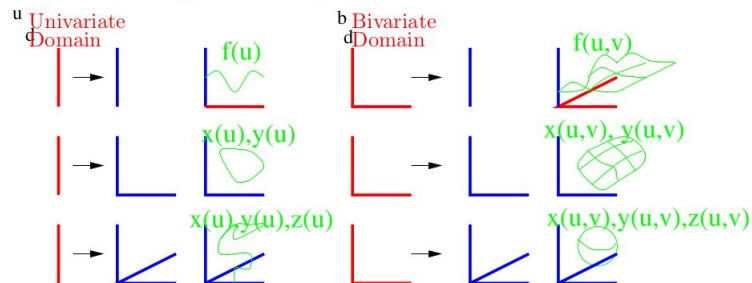
Maps and Curved Geometry

Related Concepts

- Graph of a function
- 1-Manifold ~ curve
- 2-Manifold ~ surface

- Scalar field: $\mathbb{R}^3 \rightarrow \mathbb{R}^1$ Each point in 3-space is assigned a scalar
- Vector field: $\mathbb{R}^3 \rightarrow \mathbb{R}^3$...a vector
- Tensor field: $\mathbb{R}^3 \rightarrow \mathbb{R}^{3 \times 3}$...a tensor

Domain, Range and Maps



Maps and Curved Geometry

Polynomials and Polynomial Forms

Polynomial of degree n :-
an infinitely differentiable map whose n th derivative is constant

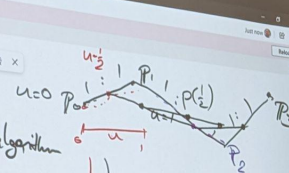
Polynomial p in of degree d in Bernstein-Bézier form (**BB-form**)

$$p := \sum_{i+j=d} c(i)B_{j,i}, \quad \text{where } B_{j,i}(u) = \frac{d!}{i!j!}u^i(1-u)^j$$

Typically, p is evaluated on the interval $[0, 1]$. This yields a polynomial piece.

Maps and Curved Geometry

When turned on by your admin, if you want to turn them off, go to the Settings menu. [Open Settings](#)



de Casteljau's algorithm

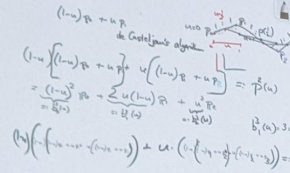
$$(1-u)p_0 + up_1$$

$$(1-u) \left[(1-u)p_0 + up_1 \right] + u \left[(1-u)p_1 + up_2 \right] =: p^2(u)$$

$$= \underbrace{(1-u)^2}_{=: b_0^2(u)} p_0 + \underbrace{2u(1-u)}_{=: b_1^2(u)} p_1 + \underbrace{u^2}_{=: b_2^2(u)} p_2 \quad b_1^3(u) = 3u(1-u)^2$$

$$(1-u) \left((1-u) \left[(1-u)p_0 + up_1 \right] + u \left[(1-u)p_1 + up_2 \right] \right) + u \cdot \left((1-u) \left[(1-u)p_1 + up_2 \right] + u \left[(1-u)p_2 + up_3 \right] \right) =: p^3(u)$$

When turned on by your admin, if you want to turn them off, go to the Settings menu. [Open Settings](#)



de Casteljau's algorithm

$$(1-u) \left[(1-u)p_0 + up_1 \right] + u \left[(1-u)p_1 + up_2 \right] = p^2(u)$$

$$= \underbrace{(1-u)^2}_{=: b_0^2(u)} p_0 + \underbrace{2u(1-u)}_{=: b_1^2(u)} p_1 + \underbrace{u^2}_{=: b_2^2(u)} p_2 \quad b_1^3(u) = 3u(1-u)^2$$

$$(1-u) \left((1-u) \left[(1-u)p_0 + up_1 \right] + u \left[(1-u)p_1 + up_2 \right] \right) + u \cdot \left((1-u) \left[(1-u)p_1 + up_2 \right] + u \left[(1-u)p_2 + up_3 \right] \right) = p^3(u)$$

ASIDE:
power form of polynomial

$$q(t) := \sum a_i t^i$$

$$\left. \begin{array}{l} q(0) = a_0 \\ q'(0) = a_1 \end{array} \right\} \begin{array}{l} q(1) = 2a_2 \\ q'(1) = n! a_n \end{array}$$

$$b_i^n(u) := \binom{n}{i} u^i (1-u)^{n-i}$$

$$\binom{n}{i} = \frac{n!}{i!(n-i)!} \quad i+j=n$$

$$= \binom{n!}{i! j!} u^i v^j \quad u+v=1$$

Maps and Curved Geometry

Piecewise polynomials in B-spline form (B-form)

Industry uses the acronym NURBS = Non-uniform rational B-spline

Uniform splines can be efficiently evaluated by subdivision.

Spline = piecewise polynomial (function).

Knots delineate the break points between polynomial pieces.

Typically some smoothness is enforced between the pieces.

A spline can be represented in Bézier form by connecting pieces in Bézier form.

A spline can be represented in B-spline form.