EXAM INSTRUCTIONS FOR PROCTORS & SITE COORDINATORS
This exam is for: COT5405 (Dr. Ranka)
(Course number and Professor)

Please return exam to: P K Manna, TA, COT5405
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FAX Number 352-392-1220 PH. Number 352-392-1200

DUE DATE: Dec 22, 2005

INSTRUCTIONS:
Last lecture or tape to be viewed before exam: ALL
Time limit for taking the exam: 120 minutes
Type of test - Open Book Closed Book X Take-Home
Or (other)

Notes Allowed: (YES/NO) No
Calculators Allowed: (YES/NO) No
Computer Allowed: (YES/NO) No
Other Specifications (crib sheets, special tables, references, etc.): No

Proctor Required: (YES/NO) YES

CERTIFICATION:
This examination has been administered in accordance with the regulations printed on the reverse side of this sheet and the conditions specified above.

________________________________________
Signature of Proctor

Date/Time Administered

PLEASE RETURN THIS SHEET WITH THE EXAMS

NOTE: Do not enclose any other exams or other materials in the envelope with this exam.

PROCTOR: PLEASE MAKE A COPY OF THE COMPETED EXAM AND FILE IT UNTIL THE GRADED EXAM IS RETURNED.
REGULATIONS FOR ADMINISTERING EXAMINATIONS

The conditions set forth on the exam instruction sheet must be rigorously observed in administering examinations in connection with the University of Florida, College of Engineering’s off-campus graduate credit courses.

1. Upon receipt, these exams must be put in a secure location for safekeeping until the exam is administered.

2. If for any reason the exam cannot be administered prior to the due date, agreement on alternative arrangements MUST be reached IN ADVANCE with the course professor.

3. The local site coordinator or a designated proctor must be present during the entire period of the examination unless stated otherwise in the exam instructions. Please verify the students identification with a picture ID (drivers license, visa, company picture ID), if unknown to you.

4. No communication is permitted among the students taking the examination unless specified in the instructions on the front of this document.

5. Questions which arise during an exam should be addressed to the professor. If circumstances prohibit access to the professor, the proctor should instruct the students to make a reasonable assumption, note the assumption on the exam, and continue with the exam OR make note of the question on a separate page and so indicate in the certification block on the exam instruction sheet.

6. The proctor should collect the exam(s), sign and date the certification on the exam instruction sheet, seal the exams together with exam instruction sheet in an envelope and return the exam(s) by postal service, courier service, or faxed immediately back to the address or number specified on the Exam Instruction page.

7. If the professors instructions for administering this exam conflict with these instructions follow the professors on the specific conflict, otherwise the instructions are to be followed.
Instructions:

1. Write neatly and legibly
2. While grading, not only your final answer but also your approach to the problem will be evaluated
3. You have to attempt all three problems (15 + 25 + 60 points). You have choices under the 3rd problem.
4. Total time for the exam is 120 minutes (2 hours)
5. You are not allowed to use a calculator for this exam

I have read carefully, and have understood the above instructions. On my honor, I have neither given nor received unauthorized aid on this examination.

Signature: ____________________________________________

Date: _____ (MM) / _____ (DD) / _________ (YYYY)
Q1. (3 * 5 = 15 Points) Complete all three parts.

You must write a very brief explanation for your answer for each question (without the justification, you will get very little credit):

a) **Tick all that apply:**
   - All the problems in NP are known to be reducible to one another
   - All the NP-complete problems are known to be reducible to one another
   - All the NP-hard problems are known to be reducible to one another
   - All of the above
   - None of the above

**Answer:** The first part is false, second and third are true (since all problems in NP are NOT reducible from one another, viz. an NP-complete problem is not reducible to a P problem)
b) Using the Venn-diagram notation of representing relations between sets, please show the relationships between the following sets: $P$, $NP$, $NP$-Hard and $NP$-Complete.

For example, to show the relationship between sets $A$, $B$ & $C$ where $A \subseteq B$ and $A \cap C = \emptyset$ and $B \cap C \neq \emptyset$, we draw:

```
B

A

C
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**Not drawing it - trivial**

c) Suppose someone proves that *some* NP-complete problems cannot be solved faster than $O(n^{289})$ time. Does that mean $P \neq NP$?

**Observation:** Note that this only gives a lower bound – this does *not* mean that he has actually given a $\Omega(n^{289})$ algorithm to solve it. For example, I can always prove that sorting cannot be done faster than $O(n)$ time (which is easily provable since for sorting, you need to read ALL the $n$ elements, which alone takes $O(n)$ time), but that does not necessarily mean I gave a $O(n)$ sorting algorithm.

False, since even if someone can come up with a $O(n^k)$ algorithm where $k > 289$, that would still mean $P = NP$. 

**Q2. (25 points) Mandatory:**

For an input \(<G,k>\) where \(G\) is an undirected graph and \(k\) is an integer, the **max-degree spanning tree** problem is to determine if there exists a spanning tree \(T\) (not necessarily minimum spanning tree) for \(G\) where the maximum degree of the vertices in \(T\) is \(k\).

(A degree of a vertex is the number of edges incident on the vertex.)

Prove that the **max-degree spanning tree** problem is NP-Complete.

*Hint: reduce it from the known NP-Complete Hamiltonian Path problem, which decides for input graph \(<G>\) if there exists a simple path covering all vertices, in \(G\).*

**Solution:** Catch is that Hamiltonian path itself is a max-2 degree spanning tree.
Q3. Attempt any 2 out of 3 subparts (2 * 30 = 60 points)

a) Prefix codes can be defined as a set of words such that no word of the set is a prefix of another word in the set. Prefix codes are usually used in encoding/decoding for message transmission. The problem is defined as follows. Given a set of messages $M_1, M_2, \ldots, M_n$ and a frequency vector $<q_1, q_2, \ldots, q_n>$ where $q_i$ indicates the relative frequency with which message $M_i$ will be transmitted, the expected cost of transmission is $\sum_{i=1}^{n} q_id_i$, where $d_i$ is the length of message $M_i$. Develop a greedy algorithm to construct an optimal binary prefix code that minimize the transmission cost. Prove the correctness of your algorithm. Apply your algorithm for coding the following letters.

Messages (each contains one letter): C, E, I, R, S, T, X
Frequency Vector: <11, 22, 16, 12, 15, 10, 14>

**Answer:** This question can be mapped to a binary tree. We are trying to find the minimal weighted external path. It’s of the same type with Huffman codes.
b) A generalized job sequencing problem is defined as follows. You are given \( n \) jobs and one processor. Each job \( i \) has associated with it a three tuple \( (p_i, d_i, t_i) \). Job \( i \) requires \( t_i \) units of processing time. If its processing can be completed by the deadline \( d_i \), then a profit \( p_i \) is earned. A feasible solution is a subset \( J \) of the \( n \) jobs such that all jobs in \( J \) can be completed by their deadlines. The value of a feasible solution \( J \) is the sum of the profits of the jobs in \( J \), or \( \sum_{i \in J} p_i \).

The objective is to select the optimal solution with the maximum value. Please design a backtracking based algorithm with a good bounding function to solve this problem.

**Answer:** This question is actually a subset-sum question. We can use a variable-sized tuples tree with a backtracking algorithm. For any node \( x \), the upper bound could be \( \sum_{i \in S_x} p_i + \sum_{j \in S_x} p_j \) where \( m = \max \{ i \mid i \in S_x \} \). The lower bound could be \( \sum_{i \in S_x} p_i \).
c) The convex hull of a set $S$ of points in the plane is defined to be the smallest convex polygon containing all the points of $S$. See figure below for an example. Develop an efficient divide and conquer algorithm for finding the convex hull of a set of two dimensional points. Derive the complexity of your algorithm.

Solution: Just refer to p.184 in the textbook