This is closed notes/book in-class exam. Electronic calculators are allowed.

You are expected to derive the best possible algorithm to solve the problems (unless indicated otherwise). Poor performance solutions (even if they are correct) may not get any credit.

Divide your solution into four parts when you are asked to design algorithm:
1. Description of the basic approach
2. Pseudo Code (or code in any high level language such as Java, C, C++)
3. Proof of Correctness
4. Complexity Analysis

You have 2 hours to finish the exam. Please write legibly.

There are 5 problems on the exam. You should attempt 4 problems -- 1, 2, 3 and one of problems 4 or 5.

Only one of the problems 4 or 5 will be graded – in case you do both, please mark one that you would like to be graded. Otherwise, we may chose to grade either of them or assign you zero points for the problem.

Good luck.

NAME: ___________________________________________________________
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1. (10 points)
Merge Sort divides the array into two nearly equal sizes, sorts each of them recursively and merges the result. Rather than separate an array into two half-size arrays for sorting, consider the following two approaches:

(a) Divide it into three subarrays of size \( \lfloor n/3 \rfloor \), \( \lceil (n + 1)/3 \rceil \) and \( \lceil (n + 2)/3 \rceil \), respectively, to sort each of these three subarrays recursively, and then merge the three sorted subarrays.

(b) Divide it into about \( \sqrt{n} \) arrays, each containing approximately \( \sqrt{n} \) elements. Describe each of the above algorithms formally and analyse their execution time.
2. (10 points)
Describe an efficient greedy algorithm that, given a set \{x_1, x_2, \ldots, x_n\} of points on the real line, determines the smallest set of unit-length closed intervals that contains all of the given points. Provide the time requirements of your algorithm and prove its correctness.
3. (15 points)  
There is a warehouse with a storage capacity of $B$ units and an initial stock of $\gamma$ units. Let $y_i$ be the quantity of product sold in each month $i$, $1 \leq i \leq n$. $P_i$ is the per-unit selling price in month $i$. Let $x_i$ be the quantity purchased in month $i$. The buying price is $c_i$ per unit. At the end of each month, the stock in hand must be no more than $B$. i.e.,

$$\gamma + \sum_{t \leq \gamma j} (x_i - y_i) \leq B, \quad 1 \leq j \leq n$$

The amount sold in each month cannot be more than the stock at the end of the previous month (new stock arrives only at the end of a month) i.e.,

$$y_i \leq \gamma + \sum_{k \leq j \neq i} (x_j - y_j), \quad 1 \leq i \leq n$$

Assume that $x_i$ and $y_i$ to be non-negative integers. The total profit derived is:

$$P_n = \sum_{j=1}^{n} (p_j y_j - c_j x_j)$$

The problem is to determine $x_j$, $y_j$ such that $P_n$ is maximized. Provide an efficient dynamic programming algorithm to determine $P_n$. Analyse the complexity of your algorithm.
4. (15 points)
The set intersection problem (SIP) is defined as follows: Given finite sets $A_1, A_2, \ldots, A_r$ and $B_1, B_2, \ldots, B_s$. Is there a set $T$ such that

$$|T \cap A_i| \geq 1 \text{ for } i = 1, 2, \ldots, r$$

and

$$|T \cap B_j| \geq 1 \text{ for } j = 1, 2, \ldots, s?$$

Show that the set intersection problem is NP-complete using a transformation from 3-CNF-SAT.
5. (15 points)
Show that one of the following problems is polynomially reducible to the other problem:

(a) You are melon-selling farmer and you have melons of different weights. A customer comes to you and asks you for exactly k pounds of uncut melons and you need to determine whether you can fulfill his request.

(b) A city road network can be described as a set of straight road segments that are connected to a set of intersections. Each police officer, if standing on an intersection can effectively monitor all the segments that are connected to that intersection. The Chief wants to determine whether he can use a force consisting of k police officers so that every segment of the city road network is monitored by at least one police officer.