Homework assignment 3,

1. A set U is $\leq_m^C$-**complete** for a class D if every set X in D is reducible to U by a $\leq_m^C$-reduction, i.e, there is a function f in the class C, such that $x \in X \iff f(x) \in U$.

   (i) Show that the set $A_{pseudocode} = \{(M, x) :$ the pseudocode M accepts $x\}$ is $\leq_m^{rec}$-complete for the class of r.e. sets.

   (ii) Give a set that is $\leq_m^{rec}$-complete for the class co-r.e., and justify your answer.

   (iii) Can a set A - that is $\leq_m^{rec}$-complete for the class of r.e. sets - be decidable. Why (not)?

2. The class $Diff$ consists of the intersections of sets in $r.e$ with the sets in $co - r.e.$. I.e, $Diff = \{A \cap B : A \in r.e., B \in co - r.e.\}$. (This is different from $r.e. \cap co - r.e.$ which we know to be the class of recursive or decidable sets).

   (i) Show that

   $$r.e. \cup co - r.e. \subseteq Diff$$

   and

   (ii) (Recalling the definition of complete given above) starting from a a set U that is known to be is $\leq_m^{rec}$-complete for r.e., construct a set, and show that it is $\leq_m^{rec}$-complete for $Diff$. Hint: how about $\{(x, y) : x \in U \wedge y \notin U\}$??

3. Read Chapter 6.4 in book. Answer 6.13 (A_{TM} is our $A_{pseudocode}$), 6.16, 6.17 (Turing-recognizable = recursively enumerable).

4. (bonus) Show that the set $A_{Halt} = \{M :$ the pseudocode M halts on input M (may or maynot accept) $\}$ is $\leq_m^{rec}$-complete for the class of r.e. sets.