Homework 1, (carries no grade - solutions will be posted in 1 week)

1. (i) Convert the following into logical notation using suggested variables. Then provide a formal proof.

   If (l)I study law, then (m)I will make a lot of money. If (a)I study archaeology, then (t)I will travel a lot. If I make money or travel a lot, then I will not be disappointed. Therefore, If I am disappointed, then I did not study law and I did not study archaeology.

   Example: Either (p)Pat did it, or (q)Quincy did it. Quincy could not (r)have been reading and done it. Quincy was reading. Therefore Pat did it.

   Solution: axioms: $p \vee q$, $\neg(r \wedge q)$, r.
   need to prove: p
   proof: $\neg(r \wedge q)$(given) $\Rightarrow (\neg r \vee \neg q)(1)$.
   $(\neg r \vee \neg q)(1) \wedge r$(given) $\Rightarrow (\neg r \wedge r)(\uparrow) \vee (\neg q \wedge r) \Rightarrow \neg q \wedge r \Rightarrow \neg q(2)$.
   $(p \vee q)$(given) $\wedge \neg q(2) \Rightarrow (p \wedge \neg q) \vee (q \wedge \neg q)(\uparrow) \Rightarrow (p \wedge \neg q) \Rightarrow p$. ♣

   (ii) Can you prove, using the above axioms, that "if I am disappointed, then I studied mathematics"? Answer yes or no, and explain why.

   (iii) Let $\Sigma = \{a, b\}$, $A = \{a, b, aa, bb, aaa, bbb\}$, $B = \{w \in \Sigma^* : \text{length}(w) \geq 2\}$, $C = \{w \in \Sigma^* : \text{length}(w) \leq 2\}$.

   (a) Determine $A \cap C$, $A \setminus C$, $C \setminus A$, $A \oplus C$ (symmetric difference), $A \cap B$, $C \cap B$, $C \cup B$, $B \setminus A$, $\Sigma^* \setminus B$, $\Sigma \setminus B$ and $\Sigma \setminus C$.

   (b) List all subsets of Σ.

   (c) If $|\Sigma| = k$ what is the size of the power set of Σ?

   (iv) Book **Exercise** 1.4 (e) (f), (i), (j).

   (v) Book **Exercise** 1.5 (b), (c), (e); 1.4 (n)

2. Book **Exercise** 1.12, 1.15, 1.16

3. Book **Problem** 1.24, 1.31, 1.37

4. Which of the following languages are regular? If the question concerns a class of languages, then either show that all the languages in the

class are regular, or that they are not regular, or exhibit one language in the class that is regular and one that is not. In all cases, justify your answer.

(i) $\{0^n$: n is the product of two or more (not neccessarily distinct) primes other than $1\}$

(ii) ADD from Book Problem 1.37

(iii) $REFLECT(L) = \{xx^{reverse} : x \in L\}$, where L is regular.

(iv) $INV - REFLECT(L) = \{x : xx^{reverse} \in L\}$, where L is regular.

5. For any language L over the alphabet $\{a, b\}$, define $1/2L$ as follows:

$$1/2L = \{w \in \{a, b\}^* | \text{ for some } x \text{ such that } |x| = |w|, wx \in L\}.$$

Show that if L is regular, so is $1/2L$.