Sample Mid Term Exam

1. **Calculus of variations:** For the functionals written below, write down the Euler-Lagrange equation. a) $E(f) = \int_0^1 f(x)f'(x)dx$, b) $E(f) = \int_0^1 f(-x)f'(x)dx$, c) $E(f) = \int_0^1 (f'')^2(x)dx$, d) $E(f) = \int_0^1 \int_0^1 (f(x,y) - d(x,y))^2 dxdy + \lambda \int_0^1 \int_0^1 [(\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial y})^2] dxdy$. For this, you need to know that for a functional of the form $S = \int_0^1 \int_0^1 L(f, f_x, f_y)dxdy$, the Euler -Lagrange equation's general form is $\frac{\partial L}{\partial f} - \frac{d}{dx}\frac{\partial L}{\partial f_x} - \frac{d}{dy}\frac{\partial L}{\partial f_y} = 0$. e) $E(f) = \int_0^1 \sqrt{(f')^2 + (f'')^2} dx$.

2. **Snakes:** Questions here will likely focus on designing energy (optimization) terms to satisfy certain constraints. a) the snake should avoid certain locations, b) allow a few high curvature points on the snake.

3. **Hough Transform:** There are several possible useful extensions of the Hough transform. a) Use the Hough transform so that you are now looking for curves of the form $ax^2 + bx + c = 0$ using pairs of edge points (x_1, y_1) and (x_2, y_2). Discuss the complexity of the vote build up and search procedure if all pairs of points are considered. What is the dimensionality of the search space? b) Extend the Hough transform for lines to equally consider edge points and edge gradients. The input information is a triple (x_i, y_i, θ_i) for $i \in \{1, \ldots N\}$. Given two parameters ϵ and δ, set up a vote counting mechanism wherein lines can pass within ϵ of the point (x_i, y_i) and have their slope at most an angle δ away from θ_i.

4. **Canny edge detection:** Expect fairly detailed and algorithmically oriented questions on Canny since you have implemented it. Possible questions are a) Clearly state how the filter size (3x3, 7x7 etc.) depends on σ. Assume that the absolute value of the first derivative must reach a value lower than some threshold Tat he width of the filter. b) Hysteresis and non-maximum suppression neighborhoods need not be restricted to one pixel. c) Reverse the order of non-maximum suppression and hysteresis and analyze the result.