Solutions to HW #3

Q7.1:

Intrinsic: Focal length of camera, field of view, aperture controls, shutter speed, CCD characteristics, geometric and illumination distortion etc. For a good and fairly complete description of some of these parameters, please visit
http://www.imaging-resource.com/PRODS/C995/C9XDAT.HTM.

Extrinsic: Rotation matrix $\boldsymbol{R}$ and translation matrix $\mathbf{T}$ between the two cameras.

Q7.3: The epipolar constraint can be best visualized by projecting a line (in 3D) that is constrained to intersect the line joining the two centers of projection onto the two image planes. You get a line in each image plane that is in correspondence by construction. Since the epipoles are the images of the projection centers, the search for correspondence can be constrained to a 1-D search.
Q7.7:

- Intrinsic and extrinsic known: Fully 3D reconstruction via triangulation.
- Only intrinsic known: 3D reconstruction up to unknown scale factor.
- No information: Reconstruction up to unknown projective transformation.

E7.9: Assume that you have two lines l_1 and l_2 in two image planes which are in correspondence. From each line and the corresponding projection center, a plane can be constructed resulting in a total of two planes. These two planes have to meet if the original two lines originated from a line in 3D. Let the two planes satisfy the equations

$$\mathrm{a}_1 x + b_1 y + c_1 z + d_1 = 0$$

and

$$a_2 x + b_2 y + c_2 z + d_2 = 0.$$

Please note that each plane will have a unique normal vector perpendicular to it. Obviously, the line of intersection of the two planes will be parallel to that vector which is perpendicular to both the normal vectors. The coefficients of the two normal vectors are (a_1, b_1, c_1) and (a_2, b_2, c_2) respectively. The cross product of the two normals will have coefficients $(b_1c_2 - b_2c_1, c_1a_2 - c_2a_1, a_1b_2 - a_2b_1)$. This vector is parallel to the line of intersection. All we need is to fix one point on this line to uniquely determine it. If the above cross product of the two normal vectors is denoted n_{cp}, and if one of the points on the line is denoted P_{cp}, then the equation of the line is $P_{cp} + wn_{cp}$ where w is arbitrary (where it is assumed that P_{cp} is written in the left camera frame or in a world frame). To determine one point on the line, note that we have two plane equations with three degrees of freedom (x, y, z). Consequently, two coordinates can be written in terms of the third. Usually, the third coordinate can be arbitrary because the line of intersection we are trying to solve for is infinitely long. This fixes the third coordinate and consequently the other two as well. Once a certain P_{cp} is fixed, the equation of the line of intersection can be obtained from the vector $P_{cp} + wn_{cp}$.
E7.10:

$$P_r = R_r P + T_r$$

and

$$P_l = R_l P + T_l.$$

From this, we get

$$P_r = R_r[R_l^{-1}(P_l - T_l)] + T_r = R_r R_l^{-1} P_l - R_r R_l^{-1} T_l + T_r = R_r R_l^{-1}(P_l - T_l + R_l R_r^{-1} T_r).$$

Since $P_r = R(P_l - T_l)$, we equate terms to get $R = R_r R_l^{-1} = R_r R_l^T$ and $T = T_l - R_l R_r^T T_r = T_l - R^T T_r$.
E7.11:

$$w_j \times w_k = (\hat{E}_j \times \hat{T}) \times (\hat{E}_k \times \hat{T}) = [(\hat{E}_j \times \hat{T})\hat{T}]\hat{E}_k - [(\hat{E}_j \times \hat{T})\hat{E}_k]\hat{T}$$

using the well known formula $A \times (B \times C) = (AC)B - (AB)C$. This can be simplifed to (since $\hat{T} \times \hat{T} = 0$),

$$w_j \times w_k = -[(\hat{E}_j \times \hat{T})\hat{E}_k]\hat{T} = \hat{T}(\hat{E}_j \times \hat{E}_k)\hat{T}.$$

Since $\hat{E}_j = \hat{R}_j \times \hat{T}$, the above can be further simplified. One can show that $\hat{E}_j \times \hat{E}_k = \hat{T}(\hat{R}_j \times \hat{R}_k)\hat{T} = \hat{T}\hat{R}_i\hat{T}$ assuming that (i, j, k) are in cyclic order. This can be back substituted above (and using the fact that $\hat{T}$ is normalized) we get

$$w_j \times w_k = -[(\hat{E}_j \times \hat{T})\hat{E}_k]\hat{T} = \hat{T}(\hat{E}_j \times \hat{E}_k)\hat{T} = \hat{T}\hat{R}_i\hat{T}.$$

We also know that $(\hat{T} \times \hat{R}_i) \times \hat{T} = \hat{R}_i - \hat{T}\hat{R}_i\hat{T}$. Since $w_i = \hat{E}_i \times \hat{T} = (\hat{T} \times \hat{R}_i) \times \hat{T}$, we get $\hat{R}_i = w_i + w_j \times w_k$.