Solutions to HW #1

Q2.1: First of all, we have to assume that the object remains focused as the focal length is varied. Assuming this, since $Zz = f^2$, and Z is fixed, we see that z varies as f^2. As f increases, z increases as f^2. Likewise, as f decreases, z decreases as f^2. Since $z + f$ is the distance of the image plane from the lens center, it increases as f increases though the proportionality is somewhat complex. And, the overall size of the image increases with f because the coordinates on the image plane scale with f. Consequently, both the x and y coordinates on the image plane scale with f whereas the distance of the image plane from the center of the lens scales in a complex manner w.r.t. f.

Q2.5: The short answer is that thresholding is a poor technique for segmentation. Since the image intensity will vary due to a number of factors including but not restricted to mutual illumination, variable reflectance, sensor noise, thresholding will almost never give us a complete object. Instead the object boundary and the interior will contain pixels which are improperly segmented. From the relation $E(p) = L(P)\frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos^4\alpha$, we see that the image irradiance $E(p)$ is proportional to the scene irradiance $L(P)$. Therefore, intensity thresholding can be expected to work only if the scene irradiance which depends on albedo is different for different objects. To ensure successful segmentation assuming the irradiance can be controlled, the scene must contain objects and background of different albedos. Then, thresholds can be chosen to segment objects corresponding to a given albedo.

E2.1: We know that $Zz = f^2$, and that $\hat{Z} = Z + f$ and $\hat{z} = z + f$. From this we get that $(\hat{Z} - f)(\hat{z} - f) = f^2 \Rightarrow \hat{Z}\hat{z} = \hat{Z}f + \hat{z}f \Rightarrow \frac{1}{f} = \frac{1}{\hat{Z}} + \frac{1}{\hat{z}}$.

E4.1: Unfortunately, this question is poorly stated since it asks you to use equation 4.3 which is very misleading. Instead of using the numerator of equation 4.3 which is just the absolute value of the right side of equation 4.2, let us focus on the left side of equation 4.2. $G(t)$ is a step edge which can be visualized as a global offset of $0.5A + 0.5Asgn(t)$ where $sgn(t)$ is an antisymmetric function taking values $sgn(t) = 1$, t>0 and $sgn(t) = -1, t \leq 0$. If the filter $f(t)$ contains any symmetric components, the integral $\int_{-W}^{W} G(-t)f(t)dt$ will not be affected since $G(t)$ is antisymmetric except for a global offset. The global offset does not make any difference. However, the denominator is affected by the symmetric component of $f(t)$ because $\int_{-W}^{W} f^2(t)dt$ will increase if $f(t)$ has any symmetric component. Since the denominator increases but the numerator does not, the SNR will

obviously become worse.

E4.2:

$$\Sigma(f) = \frac{\int_{-W}^{0} f(t)dt}{\sqrt{\int_{-W}^{W} f^2(t)dt}}, \Lambda(f') = \frac{|f'(0)|}{\sqrt{\int_{-W}^{W} (f')^2(t)dt}}.$$

Since $f_w(t) = f(t/w)$, the two measures become

$$\Sigma(f_w) = \frac{\int_{-wW}^{0} f(t/w)dt}{\sqrt{\int_{-wW}^{wW} f^2(t/w)dt}} = \frac{w\int_{-W}^{0} f(x)dx}{\sqrt{w\int_{-W}^{W} f^2(x)dx}} = \sqrt{w}\Sigma(f)$$

and

$$\Lambda(f') = \frac{|f'(0)(-1/w^2)|}{\sqrt{\int_{-wW}^{wW} (f')^2(t/w)(1/w^4)dt}} = \frac{|f'(0)|}{\sqrt{w\int_{-W}^{W} (f')^2(x)dx}} = \frac{1}{\sqrt{w}}\Lambda(f')$$