1. Prove that if \( A \) is a square matrix (i.e., \( n \times n \)) then the eigen values of \( A^T A \) and \( A A^T \) are the same.

2. Prove that if \( A \) is an \( n \times n \) real symmetric matrix, then \( \det(A) = \prod_{i=1}^{n} \lambda_i \) where \( \lambda_i \) is the \( i \)th eigen value of \( A \).

3. Prove that the set of all rational numbers (and therefore the set of all irrational numbers) is a member of the Borel \( \sigma \)-algebra on \( \mathbb{R} \). Prove that the set comprising of any singleton element: \( \{a\} \) where \( a \in \mathbb{R} \) is a member of the Borel sigma algebra.

4. Derive the Kullback-Leibler divergence between the two 1-dimensional Gaussian density functions

\[
f_1(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu_1)^2}{2\sigma^2}}
\]

and

\[
f_2(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu_2)^2}{2\sigma^2}}
\]

Note that the Gaussians differ only in their means and not their variances.