1. Prove that $\sqrt{3}$ is not a rational number.

2. Recall that Rational numbers are defined as the quotient space $(\mathbb{Z} \times (\mathbb{Z} \setminus \{0\}))/\sim$ where the equivalence class is defined as $(m_1, n_1) = (m_2, n_2)$ iff $m_1n_2 - m_2n_1 = 0$. Prove that the addition and multiplication operations, defined as $(m_1, n_1) + (m_2, n_2) = (m_1n_2 + m_2n_1, n_1n_2)$ and $(m_1, n_1) \times (m_2, n_2) = (m_1m_2, n_1n_2)$ are consistent with the above defined equivalence classes.

3. Prove that the sequence, $x_{n+1} = \frac{x_n}{2} + \frac{1}{x_n}$ with $x_0 = 1$ is a Cauchy sequence.

4. Prove that the sequence, $x_{n+1} = \frac{x_n}{2} - \frac{1}{x_n}$ with $x_0 = 1$ is not a Cauchy sequence.

5. Prove that reals are complete. In other words, prove that given any Cauchy sequence of reals, there exists a Cauchy sequence of rationals that converges to the same value.