Balanced Binary Search Trees

- height is $O(\log n)$, where $n$ is the number of elements in the tree
- AVL (Adelson-Velsky and Landis) trees
- red-black trees
- find, insert, and erase take $O(\log n)$ time
Balanced Binary Search Trees

- Indexed AVL trees
- Indexed red-black trees
- Indexed operations also take $O(\log n)$ time
Balanced Search Trees

- weight balanced binary search trees
- 2-3 & 2-3-4 trees
- AA trees
- B-trees
- B-trees
- BBST
- etc.
AVL Tree

- binary tree
- for every node $x$, define its balance factor
  
  balance factor of $x = \text{height of left subtree of } x \ - \ \text{height of right subtree of } x$

- balance factor of every node $x$ is $-1$, $0$, or $1$
Balance Factors

This is an AVL tree.
The height of an AVL tree that has $n$ nodes is at most $1.44 \log_2 (n+2)$.

The height of every $n$ node binary tree is at least $\log_2 (n+1)$. 
AVL Search Tree
insert(9)
insert(29)

RR imbalance => new node is in right subtree of right subtree of blue node (node with bf = -2)
insert(29)

RR rotation.
AVL Rotations

- RR
- LL
- RL
- LR
Red Black Trees

Colored Nodes Definition

• Binary search tree.
• Each node is colored red or black.
• Root and all external nodes are black.
• No root-to-external-node path has two consecutive red nodes.
• All root-to-external-node paths have the same number of black nodes
Example Red Black Tree
Red Black Trees

Colored Edges Definition

• Binary search tree.
• Child pointers are colored red or black.
• Pointer to an external node is black.
• No root to external node path has two consecutive red pointers.
• Every root to external node path has the same number of black pointers.
Example Red Black Tree
Red Black Tree

- The height of a red black tree that has $n$ (internal) nodes is between $\log_2(n+1)$ and $2\log_2(n+1)$.
- C++ STL Class map => red black tree
Graphs

• $G = (V,E)$
• $V$ is the vertex set.
• Vertices are also called nodes and points.
• $E$ is the edge set.
• Each edge connects two different vertices.
• Edges are also called arcs and lines.
• Directed edge has an orientation $(u,v)$. 

\[ u \rightarrow v \]
Graphs

- Undirected edge has no orientation \((u,v)\).

- Undirected graph \(\Rightarrow\) no oriented edge.

- Directed graph \(\Rightarrow\) every edge has an orientation.
Undirected Graph

Diagram of an undirected graph with nodes labeled from 1 to 11.
Directed Graph (Digraph)
Applications—Communication Network

- Vertex = city, edge = communication link.
Driving Distance/Time Map

• Vertex = city, edge weight = driving distance/time.
Street Map

- Some streets are one way.
Complete Undirected Graph

Has all possible edges.

\[ n = 1 \quad n = 2 \quad n = 3 \quad n = 4 \]
Number Of Edges—Undirected Graph

• Each edge is of the form \((u,v)\), \(u \neq v\).

• Number of such pairs in an \(n\) vertex graph is \(n(n-1)\).

• Since edge \((u,v)\) is the same as edge \((v,u)\), the number of edges in a complete undirected graph is \(n(n-1)/2\).

• Number of edges in an undirected graph is \(\leq n(n-1)/2\).
Number Of Edges—Directed Graph

- Each edge is of the form $(u,v)$, $u \neq v$.
- Number of such pairs in an $n$ vertex graph is $n(n-1)$.
- Since edge $(u,v)$ is not the same as edge $(v,u)$, the number of edges in a complete directed graph is $n(n-1)$.
- Number of edges in a directed graph is $\leq n(n-1)$. 
Vertex Degree

Number of edges incident to vertex.

$\text{degree}(2) = 2$, $\text{degree}(5) = 3$, $\text{degree}(3) = 1$
Sum of degrees $= 2e$ (e is number of edges)
In-Degree Of A Vertex

in-degree is number of incoming edges

indegree(2) = 1, indegree(8) = 0
Out-Degree Of A Vertex

- out-degree is number of outbound edges
- outdegree(2) = 1, outdegree(8) = 2
Sum Of In- And Out-Degrees

Each edge contributes 1 to the in-degree of some vertex and 1 to the out-degree of some other vertex.

Sum of in-degrees = sum of out-degrees = e, where e is the number of edges in the digraph.