Undirected Graph
Directed Graph
Graph Representation

- Node objects and pointers
- Adjacency Matrix
- Adjacency List
Graph Problems

- Minimum Spanning Tree
- Shortest Path
  - Single source, single destination
  - Single source, all destination
  - All pairs
- Traveling Salesman
- Vertex Cover
- …
Dijkstra’s Algorithm

\[ d[s] \leftarrow 0 \]

for each \( v \in V - \{s\} \)
\[ d[v] \leftarrow \infty \]

\( S \leftarrow \emptyset \)
\( Q \leftarrow V \)
while \( Q \neq \emptyset \)
\[ u \leftarrow \text{Extract-Min}(Q) \]
\( S \leftarrow S \cup \{u\} \)
for each \( v \in Adj[u] \)
do if \( d[v] > d[u] + w(u,v) \)
then \( d[v] \leftarrow d[u] + w(u,v) \)

\( Q \) is a priority queue maintaining \( V - S \)
\( v : \) vertex
\( s : \) source
\( V : \) set of all vertices in graph
\( S : \) set of vertices
Dijkstra’s Algorithm

\[ Q: \begin{array}{cccccc}
A & B & C & D & E \\
0 & \infty & \infty & \infty & \infty
\end{array} \]

\[ S: \{\} \]
Dijkstra’s Algorithm

\[ S: \{ A \} \]
Dijkstra’s Algorithm

\[ S: \{A, C\} \]
Dijkstra’s Algorithm

\[ Q: \begin{array}{cccccc}
A & B & C & D & E \\
0 & \infty & \infty & \infty & \infty \\
10 & 3 & \infty & \infty & \infty \\
7 & 11 & 5 & & \\
\end{array} \]

\[ S: \{A, C, E\} \]
Dijkstra’s Algorithm

\[ S: \{A, C, E, B\} \]
Dijkstra’s Algorithm

Q:

<table>
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<th></th>
<th>A</th>
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<th>C</th>
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$S: \{A, C, E, B, D\}$
Heuristics

- Greedy methods are heuristic methods.
- A heuristic, in the context of computing, is a rule of thumb for solving a problem without the exhaustive application of an algorithm.
- They are used to reduce the problem size/complexity to make solving easier.
- Heuristics do not guarantee an optimal solution; they do not guarantee a solution!
NP-hard Problems

- These are problems for which no polynomial time algorithms is known to exist
- NP stands for Non-deterministic Polynomial-time
- NP is the set of problems who’s solutions can be verified in polynomial time (P is a subset of NP)
- A problem $\Pi$ such that for any $\Pi' \in NP$ we have $\Pi' \leq \Pi$, is called NP-hard
- An NP-hard problem that belongs to NP is called NP-complete
The Millennium Prize are seven problems in mathematics that were stated by the Clay Mathematics Institute in 2000.

A correct solution to any of the problems results in a $1M prize.
Millennium Prize Problems

1. P versus NP
2. The Hodge conjecture
3. The Poincaré conjecture (proven)
4. The Reimann hypothesis
5. Yang-Mills existence and mass gap
6. Navier-Stokes existence and smoothness
7. The Birch and Swinnerton-Dyer conjecture